

General Certificate of Education  
June 2006  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 1**

**MPC1**

Monday 22 May 2006 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

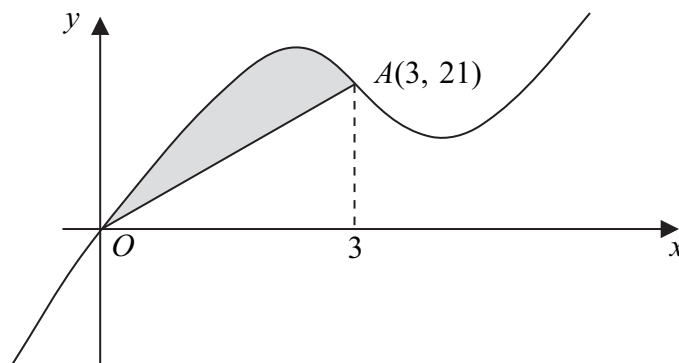
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Answer **all** questions.

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- 1 The point  $A$  has coordinates  $(1, 7)$  and the point  $B$  has coordinates  $(5, 1)$ .
- (a) (i) Find the gradient of the line  $AB$ . (2 marks)
- (ii) Hence, or otherwise, show that the line  $AB$  has equation  $3x + 2y = 17$ . (2 marks)
- (b) The line  $AB$  intersects the line with equation  $x - 4y = 8$  at the point  $C$ . Find the coordinates of  $C$ . (3 marks)
- (c) Find an equation of the line through  $A$  which is perpendicular to  $AB$ . (3 marks)
- 2 (a) Express  $x^2 + 8x + 19$  in the form  $(x + p)^2 + q$ , where  $p$  and  $q$  are integers. (2 marks)
- (b) Hence, or otherwise, show that the equation  $x^2 + 8x + 19 = 0$  has no real solutions. (2 marks)
- (c) Sketch the graph of  $y = x^2 + 8x + 19$ , stating the coordinates of the minimum point and the point where the graph crosses the  $y$ -axis. (3 marks)
- (d) Describe geometrically the transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 + 8x + 19$ . (3 marks)
- 3 A curve has equation  $y = 7 - 2x^5$ .
- (a) Find  $\frac{dy}{dx}$ . (2 marks)
- (b) Find an equation for the tangent to the curve at the point where  $x = 1$ . (3 marks)
- (c) Determine whether  $y$  is increasing or decreasing when  $x = -2$ . (2 marks)
- 4 (a) Express  $(4\sqrt{5} - 1)(\sqrt{5} + 3)$  in the form  $p + q\sqrt{5}$ , where  $p$  and  $q$  are integers. (3 marks)
- (b) Show that  $\frac{\sqrt{75} - \sqrt{27}}{\sqrt{3}}$  is an integer and find its value. (3 marks)

5 The curve with equation  $y = x^3 - 10x^2 + 28x$  is sketched below.



The curve crosses the  $x$ -axis at the origin  $O$  and the point  $A(3, 21)$  lies on the curve.

- (a) (i) Find  $\frac{dy}{dx}$ . (3 marks)
- (ii) Hence show that the curve has a stationary point when  $x = 2$  and find the  $x$ -coordinate of the other stationary point. (4 marks)
- (b) (i) Find  $\int (x^3 - 10x^2 + 28x) dx$ . (3 marks)
- (ii) Hence show that  $\int_0^3 (x^3 - 10x^2 + 28x) dx = 56\frac{1}{4}$ . (2 marks)
- (iii) Hence determine the area of the shaded region bounded by the curve and the line  $OA$ . (3 marks)

6 The polynomial  $p(x)$  is given by  $p(x) = x^3 - 4x^2 + 3x$ .

- (a) Use the Factor Theorem to show that  $x - 3$  is a factor of  $p(x)$ . (2 marks)
- (b) Express  $p(x)$  as the product of three linear factors. (2 marks)
- (c) (i) Use the Remainder Theorem to find the remainder,  $r$ , when  $p(x)$  is divided by  $x - 2$ . (2 marks)
- (ii) Using algebraic division, or otherwise, express  $p(x)$  in the form

$$(x - 2)(x^2 + ax + b) + r$$

where  $a$ ,  $b$  and  $r$  are constants.

(4 marks)

**Turn over for the next question**

**Turn over ►**

7 A circle has equation  $x^2 + y^2 - 4x - 14 = 0$ .

(a) Find:

(i) the coordinates of the centre of the circle; (3 marks)

(ii) the radius of the circle in the form  $p\sqrt{2}$ , where  $p$  is an integer. (3 marks)

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. (3 marks)

(c) A line has equation  $y = 2k - x$ , where  $k$  is a constant.

(i) Show that the  $x$ -coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0 \quad (3 \text{ marks})$$

(ii) Find the values of  $k$  for which the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0$$

has equal roots. (4 marks)

(iii) Describe the geometrical relationship between the line and the circle when  $k$  takes either of the values found in part (c)(ii). (1 mark)

**END OF QUESTIONS**